Student Name: _____

Student Number: | | | | | | | | | |

Teacher Name: _____

ABBOTSLEIGH

2023

HIGHER SCHOOL CERTIFICATE

Assessment 4

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen only.
- **NESA approved** calculators may be used.
- **NESA approved** reference sheet is provided.
- Make sure your HSC candidate number is on the front of each question.
- Answer Section I Multiple-Choice questions on the answer sheet provided.
- Answer Section II questions in the space provided.
- There is a spare page for extra working at the end of each question in Section II.
- In Questions 11 - 15, show relevant mathematical reasoning and/ or calculations

Total Marks – 100

- Attempt Sections I and II

Section I — Pages 1-8

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II — Pages 9 - 44

90 marks

- Attempt Questions 11 - 15
- Allow about 2 hour 45 minutes for this section.

Outcomes to be assessed:

Mathematics Advanced

Preliminary:

A student

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

MA11-6 manipulates, solves expressions using the logarithmic & index laws, uses logarithms, exponential functions to solve practical problems

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

MA11-8 uses appropriate technology to investigate, organise, model and interpret information in a range of contexts

MA11-9 provides reasoning to support conclusions which are appropriate to the context

HSC:

A student

MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts

MA12-2 models and solves problems and makes informed decisions using mathematical reasoning and techniques

MA12-3 applies calculus techniques to model and solve problems

MA12-4 applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems

MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

MA12-6 applies appropriate differentiation methods to solve problems

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

MA12-8 solves problems using appropriate statistical processes

MA12-9 chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use

MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Section I (10 marks)

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you've made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

(A) ⊗ (B) ⊗ (C) ○ (D) ○

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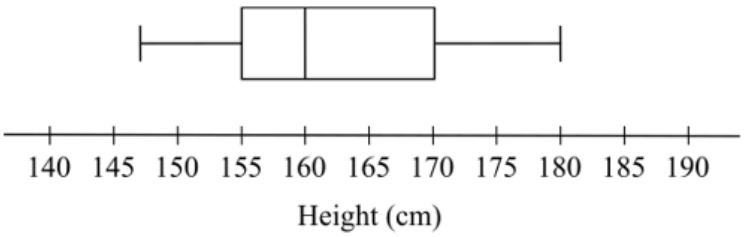
1. The heights of students in a class are represented in the boxplot below.



A new student, who will be the shortest member, is joining the class.

What is the minimum height the new student can be to **NOT** be classified as an outlier?

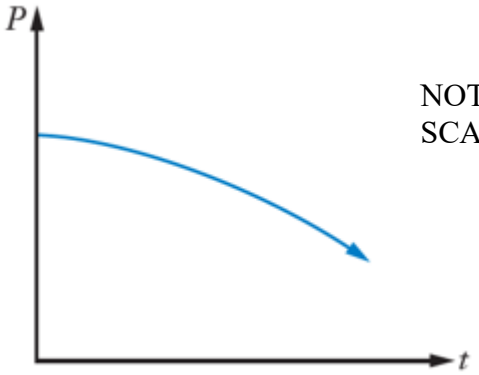
A. 127.5 cm

B. 132.5 cm

C. 137.5 cm

D. 142.5 cm

2. The population, P, of a town is shown over time, t, by the graph below.



NOT TO SCALE

Which statement best describes the population of the town?

A. The population is increasing at an increasing rate.

B. The population is decreasing at a decreasing rate.

C. The population is decreasing at an increasing rate.

D. The population is increasing at a decreasing rate.

3. Consider the two sets $A = \{5,7,9\}$ and $B = \{11,12,13\}$.

A number is chosen at random from each set.

What is the probability that the sum of the two numbers selected is greater than or equal to 21?

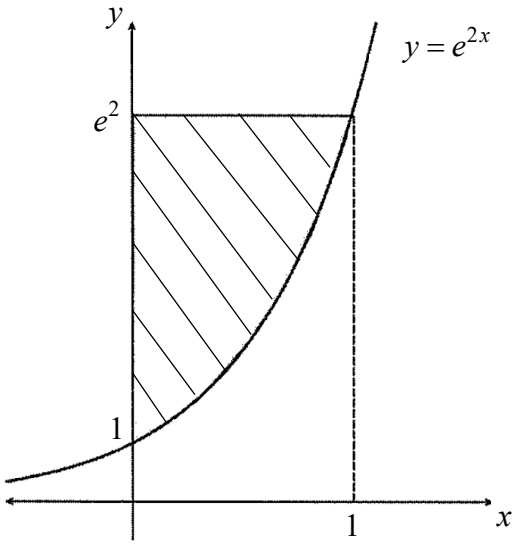
A. $\frac{1}{9}$

B. $\frac{1}{6}$

C. $\frac{2}{9}$

D. $\frac{1}{3}$

4. Which of the following would give the correct value of the shaded area shown in the diagram below?



NOT TO SCALE

A. $\int_0^1 e^{2x}\,dx$

B. $\int_0^1 \frac{1}{2}\ln x\,dx$

C. $\int_1^{e^2} e^{2y}\,dy$

D. $\int_1^{e^2} \frac{1}{2}\ln y\,dy$

5. What is the domain of the function $f(x) = \ln(3-x) + \sqrt{x-2}$?

A. $(2,3)$

B. $[2,3)$

C. $(2,3]$

D. $[2,3]$

6. An infinite geometric series has a first term of 12 and a limiting sum of 36.

What is the common ratio?

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

7. What is the correct expression for the indefinite integral $\int \frac{5x}{x^2-3}\,dx$

A. $\frac{2}{5}\ln\left|x^2-3\right|+C$

B. $\frac{5}{2}\ln\left|x^2-3\right|+C$

C. $2\ln\left|x^2-3\right|+C$

D. $\frac{1}{5}\ln\left|x^2-3\right|+C$

8. The table below shows the values of the functions $f(x)$ and $g(x)$ for various values of x.

x	1	2	3	4	5
$f(x)$	3	4	5	1	2
$g(x)$	5	3	2	1	4

What is the value of $f(g(4))$?

A. 1

B. 2

C. 3

D. 4

9. Which transformations listed are required to obtain the graph of $y = x^2 + \frac{1}{2}x - 3$ from the graph of $y = 4x^2 + x$?

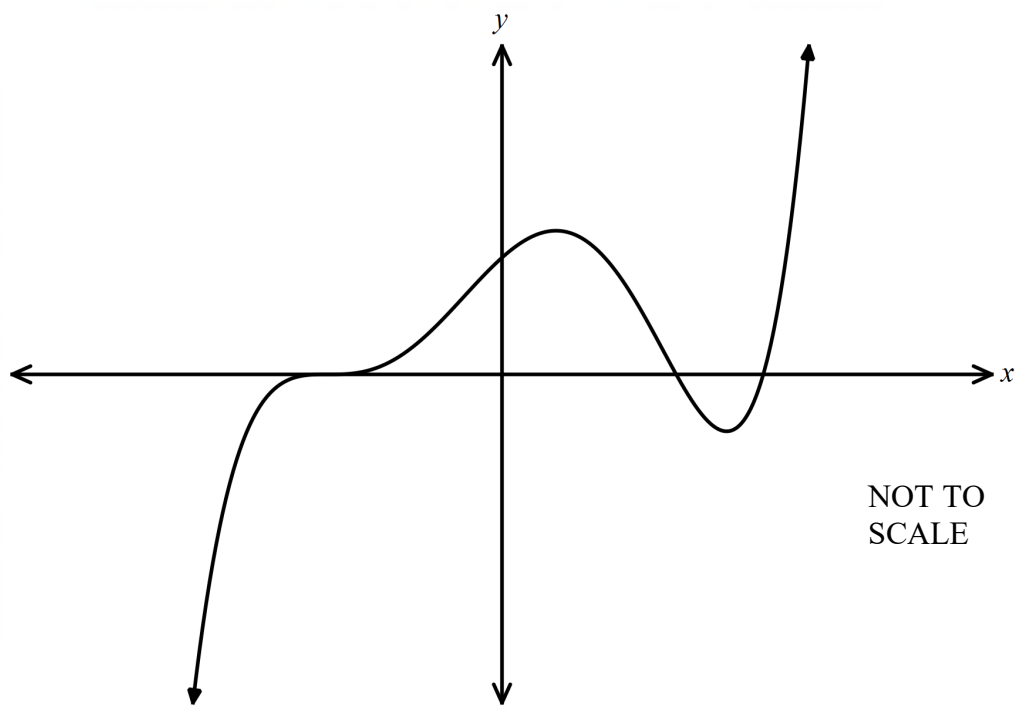
A. Horizontal dilation by a factor of 2; vertical translation of 3 units upwards

B. Horizontal dilation by a factor of 2; vertical translation of 3 units downwards

C. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units upwards

D. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units downwards

10. The graph of $y = f'(x)$ is shown below.



How many inflection points does $y = f(x)$ have?

A. 0

B. 1

C. 2

D. 3

End of Section I

Student Number: | | | | | | | | | |

2023 HSC Course

Assessment Task 4

Mathematics Advanced

Section II
90 marks

Attempt Questions 11-15
Allow about 2 hour and 45 minutes for this section

Answer the questions in the spaces provided.

These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

Question 11 (18 marks) **Marks**

(a) What is the derivative of $\sqrt{3+x^2}$? **2**

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Question 11 continues on next page

Question 11 (continued)

(b) A bakery shop sells bread rolls for \$0.60 each. The cost, C dollars, of making n bread rolls is given by $C = 200 + 0.2n$.

(i) How much does every 100 bread rolls made add to the cost of production? **1**

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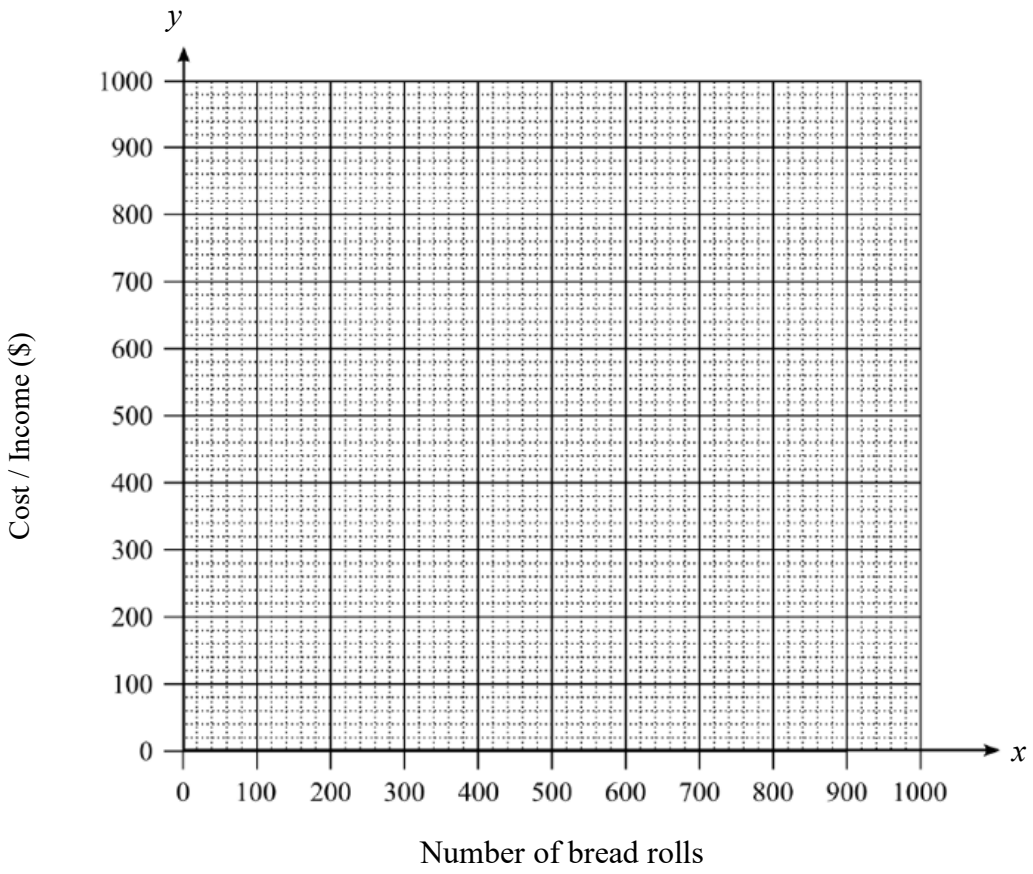
(ii) On the grid below, draw the graphs of the cost, C, and the income, I. **2**



Hence, find the number of bread rolls that must be sold to break even.

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(iii) How many bread rolls must be sold to make a profit of \$120? **2**

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Question 11 continues on next page

Question 11 (continued)

(c) Let $f(x) = \dfrac{x}{x^2+1}$.

(i) Find $f'(x)$. **2**

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(ii) Hence, or otherwise, find the equation of the tangent to the graph of $y = f(x)$ at the point where $x = 2$, in general form. **3**

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Question 11 continues on next page

(d) The probability distribution of a random variable X is shown below. **1**

x	0	1	2	3	4
$\mathrm{P}(X = x)$	k	$2k$	$3k$	$2k$	k

What is the value of k?

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(e) Given that $\ln a - \ln b = \ln(a-b)$, where $a > b > 1$, show that $a = \dfrac{b^2}{b-1}$. **2**

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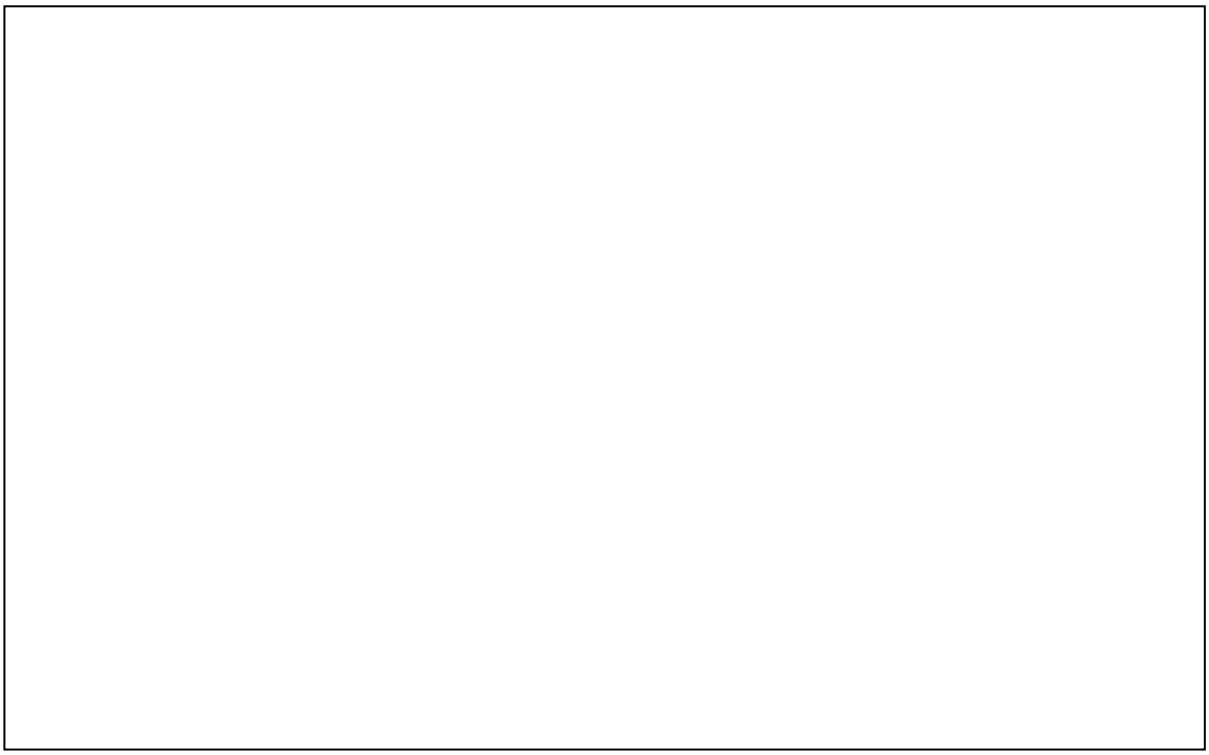
Question 11 continues on next page

Question 11 (continued)

(f) (i) Sketch the graph of $y = 2|x+1|$. **1**



(ii) Hence, or otherwise, find $\int_{-4}^{0} 2|x+1| \, dx$. **2**

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End of Question 11

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 12 (18 marks)

- For questions in Question 12, your responses should include relevant reasoning and/or calculations.

(a) Let $f'(x) = x^2 + kx$. **3**

The line $y = 2x + 1$ is a tangent to the graph of $y = f(x)$ at the point where $x = -1$.

Find $f(x)$.

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Question 12 continues on next page

Question 12 (continued)

(b) Helga is training for a marathon. Her training includes a run every Sunday, starting with a run of 3 km on the first Sunday. Each Sunday she increases the length of her run from the previous Sunday by 2 km.

(i) Show that on the 5^{th} Sunday of training, she runs 11 km. **1**

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(ii) Find a simplified expression for length of her training run on the n th Sunday. **1**

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(iii) On the n th Sunday, Helga runs 43 km. Find the value of n. **1**

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(iv) Find the total distance she runs on Sundays, in 15 weeks of training. **2**

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Question 12 continues on next page

Question 12 (continued)

(c) The frequency distribution table and cumulative frequency distribution table below show the distribution of the heights of trees in a garden. Some of the values are missing.

Height (m)	Frequency		Height less than	Cumulative Frequency
0.1-0.4			0.45	2
0.5-0.8	6		0.85	
0.9-1.2			1.25	15
1.3-1.6	11		1.65	
1.7-2.0			2.05	40
2.1-2.4	3		2.45	

(i) Complete the tables by filling in the missing values. **2**

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(ii) What is the modal class? **1**

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(iii) If a tree is randomly selected, find the probability the height is less than 1.25 m but not less than 0.45 m. **1**

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Question 12 continues on next page

Question 12 (continued)

(d) Let $f(x) = x^3 - x^2 - x$. Find the stationary points on the graph of $y = f(x)$ and determine their nature. **3**

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Question 12 continues on next page

Question 12 (continued)

(e) Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.55$ and $P(B \mid A) = 0.6$.

(i) Determine whether A and B are independent events. **1**

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(ii) Find $P(A \cup B)$. **2**

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End of Question 12

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 13 (18 marks)

- For questions in Question 13, your responses should include relevant reasoning and/or calculations.

(a) Consider the geometric series below. **2**

$$\left(\frac{2}{x+3}\right) + \left(\frac{2}{x+3}\right)^2 + \left(\frac{2}{x+3}\right)^3 + \ldots$$

Show that the series will have a limiting sum when $x = 2,$ but not when $x = -2.$

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Question 13 continues on next page

Question 13 (continued)

(b) For what values of x, in the interval $0 \le x \le 2\pi$, does the line $y = -1$ intersect the graph of $y = 2\cos 3x$? **3**

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Question 13 continues on next page

Question 13 (continued)

(c) Consider the function $f(x) = 3x - \sin x$.

(i) Show that $f(x)$ is an odd function. **1**

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(ii) Given that $f(x) \geq 0$ for $0 \leq x \leq \pi$, find the area bounded by the curve $y = f(x)$ and the $x-$axis, from $x = -\pi$ to $x = \pi$, leaving your answer in exact form. **2**

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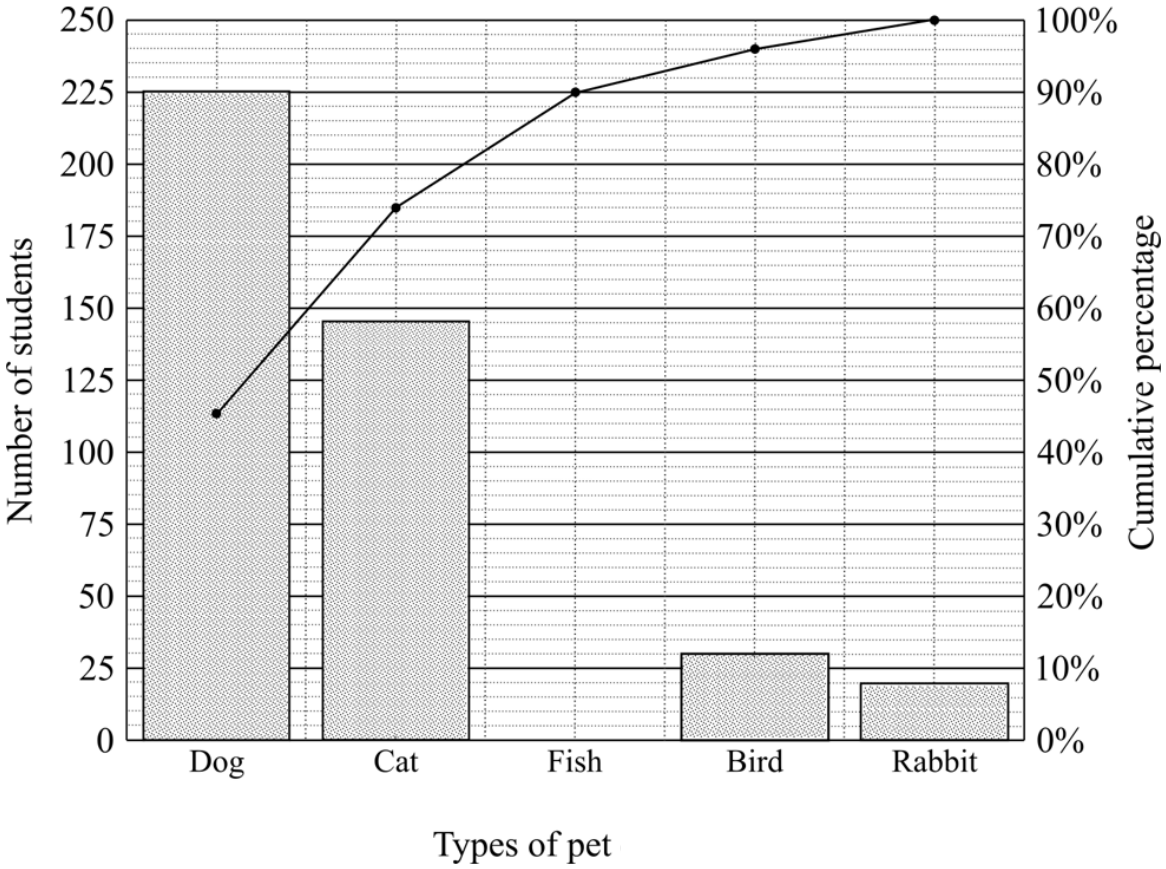
Question 13 continues on next page

Question 13 (continued)

(d) A group of students was surveyed and asked what their favourite type of pet is. The Pareto Chart shows the data collected. The column representing the number of students whose favourite pet is a fish has been removed.



(i) How many students said their favourite pet is a dog or a cat? **1**

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(ii) How many students said their favourite pet is a fish? **2**

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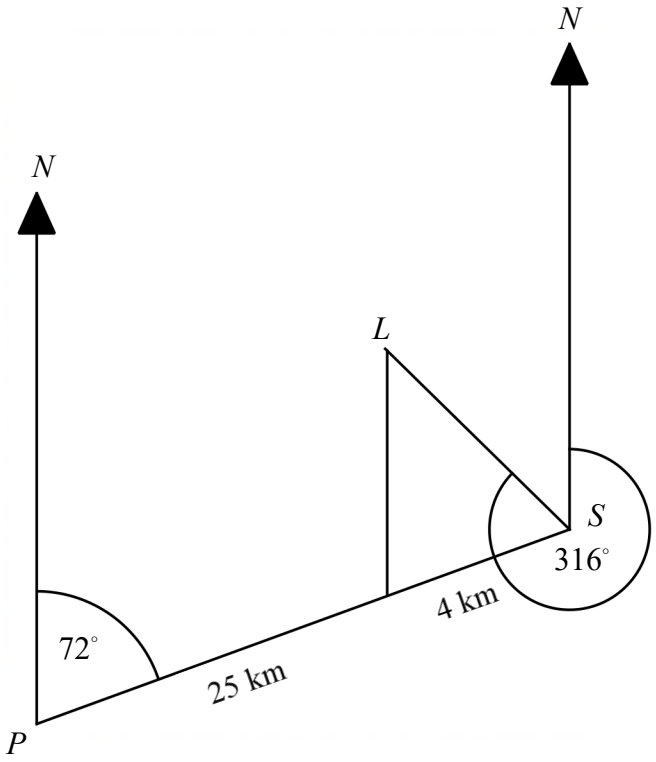
Question 13 continues on next page

Question 13 (continued)

(e) A ship (S) leaves Port (P) travelling on a bearing of 072°.

After travelling 25 kilometres, the ship is due South of a lighthouse (L).

The ship continues on this bearing for a further 4 kilometres, then measures the bearing of the lighthouse to be 316°.



NOT TO SCALE

(i) Show that $\angle PSL = 64^\circ$. **1**

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(ii) Calculate the distance LS from the ship to the lighthouse at this time. **2**

Give your answer correct to 1 decimal place.

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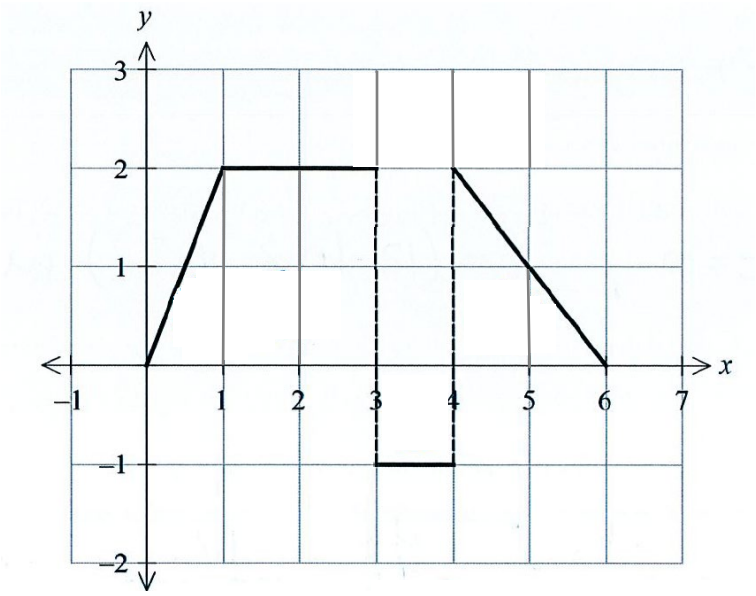
Question 13 continues on next page

Question 13 (continued)

(f) Consider the function $f(x)$ shown below.



Evaluate the following integrals.

(i) $\int_0^6 f(x)\ dx$ **1**

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(ii) $\int_0^4 \left[f(x) - 2\right]\ dx$ **2**

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Question 13f continues on next page

Question 13f (continued)

(iii) $\int_5^6 f'(x)\ dx$ **1**

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End of Question 13

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 14 (18 marks)

- For questions in Question 14, your responses should include relevant reasoning and/or calculations.

(a) Find $\int \cos x\left(1-\sin^2 x\right)\, dx$. **3**

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Question 14 continues on next page

Question 14 (continued)

(b) The sum of the first n terms of an arithmetic series is given by the formula **3**

$S_n = 3n^2 - 17n,$ where $n > 0.$ Find an expression for the nth term of the series.

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Question 14 continues on next page

Question 14 (continued)

(c) The population of parrots, P, is modelled by the function $P = P_0 e^{-kt}$, where t is time in years since May 2004.

In May 2004, there were 2500 parrots and by May 2014 the population had decreased to 1750.

(i) Show that $P_0 = 2500$. **1**

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(ii) Find the value of k. (Answer correct to four decimal places). **2**

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Question 14c continues on next page

Question 14c (continued)

(iii) If the population continues to decrease in this manner, what will be the expected population in May 2024? Answer to the nearest whole number. **2**

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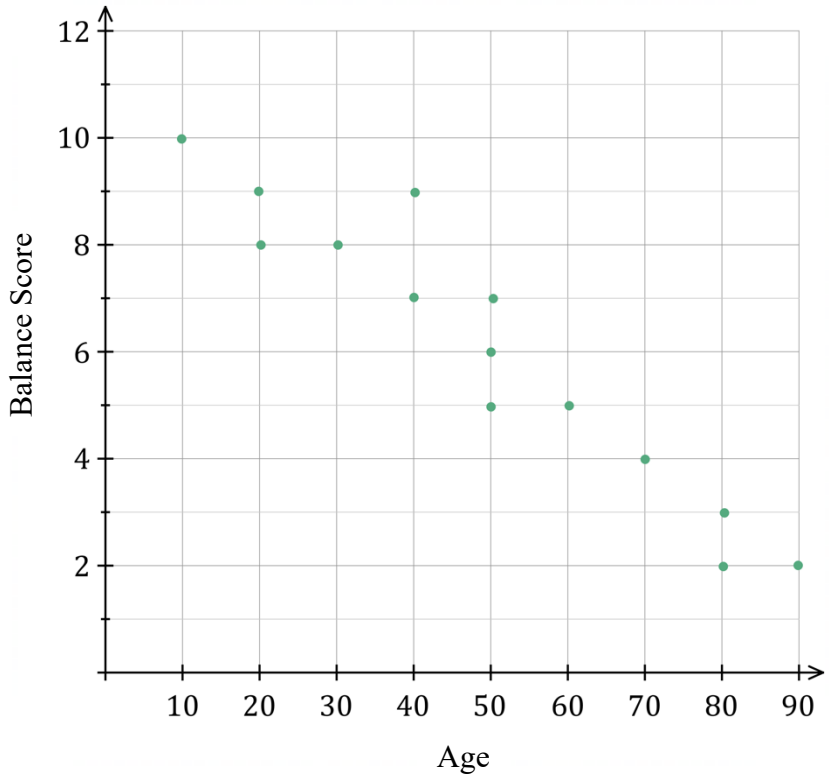
Question 14 continues on next page

Question 14 (continued)

(d) A scatter plot below shows the relationship between Age and Balance Score.



(i) The correlation coefficient is -0.955. Describe the association between Age and Balance Score with reference to the correlation. **1**

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(ii) The least squares regression line for this data is $y = 11.1249 - 0.1025x$. **1**
Using this regression line, predict the Balance Score of a 65 year old.

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(iii) Comment on whether your answer in part (ii) is reliable. **1**

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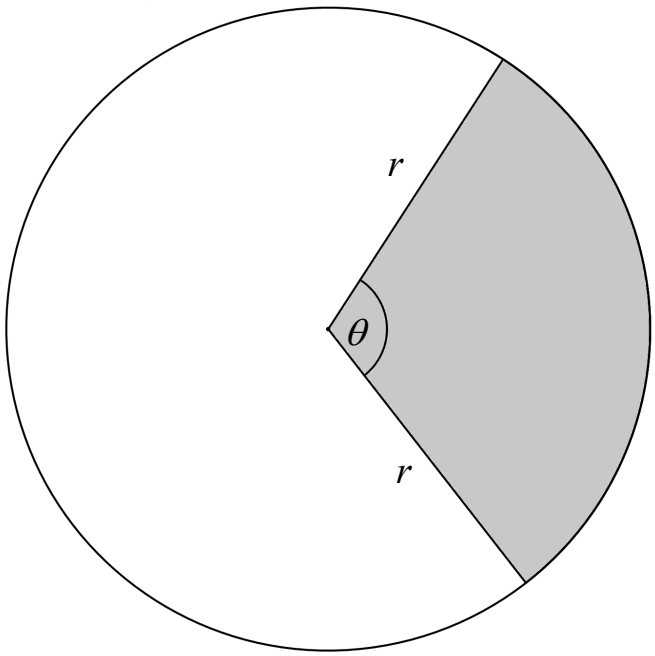
Question 14 continues on next page

Question 14 (continued)

(e) The diagram below shows a shaded sector in a circle with radius r and centre O. **4**
The sector subtends an angle of θ at the centre of the circle.
The area of the sector is $49\ \text{cm}^2$.



NOT TO SCALE

Show that $\theta = \dfrac{98}{r^2}$ and hence show that the minimum perimeter of the sector will occur when the arc length of the sector is equal to the diameter of the circle.

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Question 14e continues on next page

Question 14e (continued)

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End of Question 14

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 15 (18 marks)

- For questions in Question 15, your responses should include relevant reasoning and/or calculations.

(a) As a particle moves, its velocity, in metres per second, is described by the equation

$$v(t) = -2t^2 + 2t + 4$$

where $t \geq 0$ is the time in seconds. The particle is initially 4 metres right of the origin.

(i) Find the time taken for the particle to reach its maximum velocity. **2**

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Question 15a continues on next page

Question 15a (continued)

(ii) Find the position of the particle when it first comes to rest. **3**

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Question 15a continues on next page

Question 15a (continued)

(iii) Find the distance travelled by the particle in the first 3 seconds. **2**

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Question 15 continues on next page

(b) The point $A(4,21)$ lies on the graph of $y = g(x)$. **3**

It is known that the graph of $y = g(x)$ is obtained from transforming the graph of $y = f(x)$ such that $g(x) = -2f(4x+4)+1$.

Find the coordinates of the point on the graph of $y = f(x)$ which is mapped to the point A.

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Question 15 continues on next page

Question 15 (continued)

(c) The function $y = f(x)$ is defined as **2**

$$f(x) = \begin{cases} 1 & \text{for} \quad -3 \le x \le 0 \\ -x^2 + 1 & \text{for} \quad x > 0 \end{cases}$$

Determine whether $y = f(x)$ is both continuous **and** differentiable at $x = 0$.
Justify your answer with appropriate calculations.

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Question 15 continues on next page

Question 15 (continued)

(d) On a given day, the height of the water in a river is modelled by the function

$$h(t) = 5 + 2\sin\left(\frac{\pi t}{4}\right),$$

where h is the height of the water, in metres, and t is the time, in hours, after 12 am.

(i) What is the height of the water at 12 am? **1**

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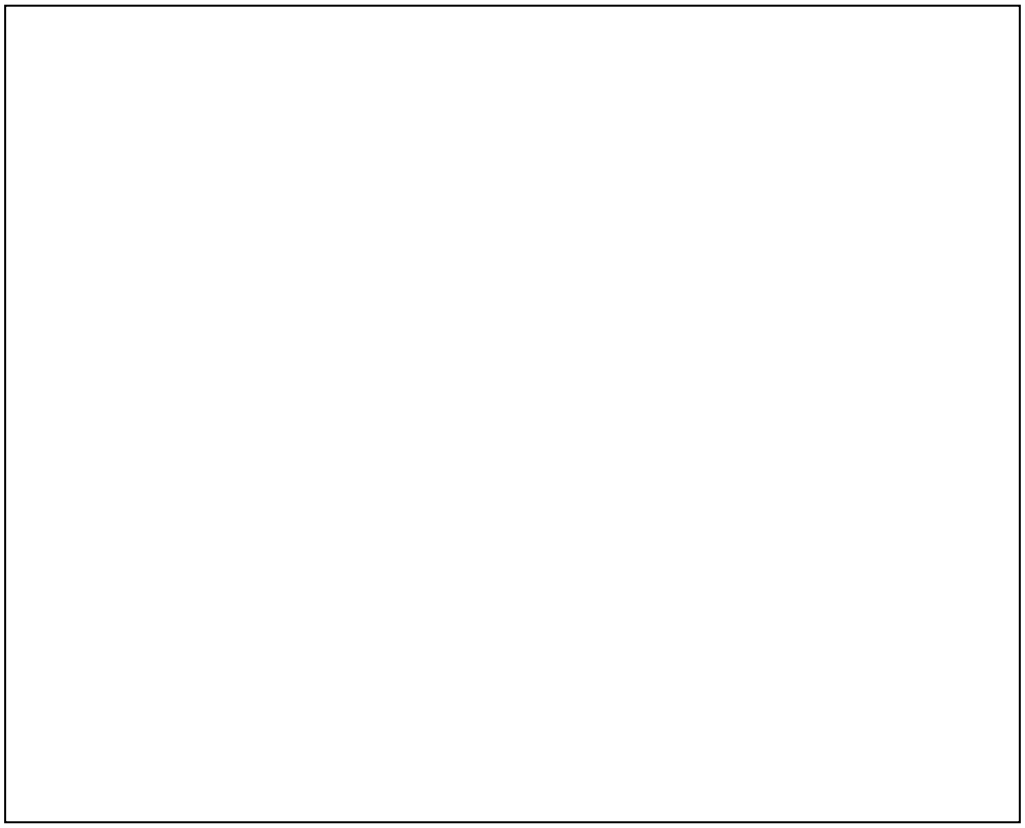
(ii) Sketch the graph of $h(t) = 5 + 2\sin\left(\frac{\pi t}{4}\right)$, in the domain $[0,16]$. **2**



Question 15d continues on next page

Question 15d (continued)

(iii) A family decides to go on a picnic by the river from 12 pm to 2 pm. **3**
It is only safe to swim in the river if the height of the water is less than 4 metres.

When is the earliest time the family can swim in the river after 12 pm?
Give your answer correct to the nearest minute.

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End of Paper

Section II Extra writing space

Student Number:

Question Number:

Student Name: Solutions

Student Number:

Teacher Name:

ABBOTSLEIGH

2023

HIGHER SCHOOL CERTIFICATE

Assessment 4

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen only.
- **NESA approved** calculators may be used.
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- There is a spare page for extra working at the end of each question in Section II.
- In Questions 11 - 15, show relevant mathematical reasoning and/ or calculations

Total Marks – 100

- Attempt Sections I and II

Section I — Pages 1-8

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II — Pages 9 - 44

90 marks

- Attempt Questions 11 - 15
- Allow about 2 hour 45 minutes for this section.

Outcomes to be assessed:

Mathematics Advanced

Preliminary:

A student

MA11-1 uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems

MA11-2 uses the concepts of functions and relations to model, analyse and solve practical problems

MA11-3 uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes

MA11-4 uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities

MA11-5 interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems

MA11-6 manipulates, solves expressions using the logarithmic & index laws, uses logarithms, exponential functions to solve practical problems

MA11-7 uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions

MA11-8 uses appropriate technology to investigate, organise, model and interpret information in a range of contexts

MA11-9 provides reasoning to support conclusions which are appropriate to the context

HSC:

A student

MA12-1 uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts

MA12-2 models and solves problems and makes informed decisions using mathematical reasoning and techniques

MA12-3 applies calculus techniques to model and solve problems

MA12-4 applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems

MA12-5 applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs

MA12-6 applies appropriate differentiation methods to solve problems

MA12-7 applies the concepts and techniques of indefinite and definite integrals in the solution of problems

MA12-8 solves problems using appropriate statistical processes

MA12-9 chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use

MA12-10 constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Section I (10 marks)

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you've made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

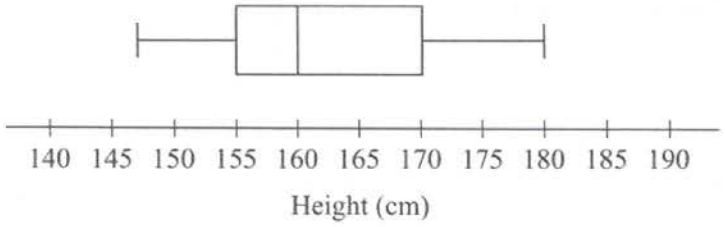
If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ←

(A) ⊗ (B) ⊗ (C) ○ (D) ○

---

1. The heights of students in a class are represented in the boxplot below.



A new student, who will be the shortest member, is joining the class.

What is the minimum height the new student can be to **NOT** be classified as an outlier?

$Q_1 - 1.5 \times IQR$

$IQR = 170 - 155$

$= 15$

$Q_1 = 155$

$155 - 1.5 \times 15 = 132.5$

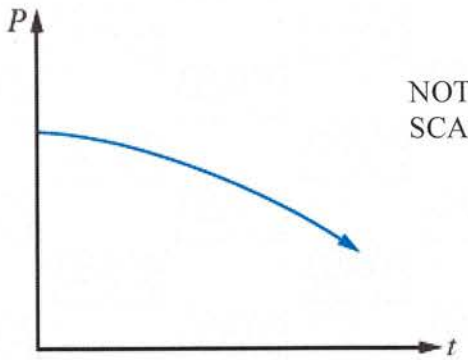
A. 127.5 cm

(B.) 132.5 cm

C. 137.5 cm

D. 142.5 cm

2. The population, P, of a town is shown over time, t, by the graph below.



NOT TO SCALE

Which statement best describes the population of the town?

A. The population is increasing at an increasing rate.

B. The population is decreasing at a decreasing rate.

C. The population is decreasing at an increasing rate. $\frac{dP}{dt} < 0$, $\frac{d^2P}{dt^2} < 0$

D. The population is increasing at a decreasing rate.

3. Consider the two sets $A = \{5,7,9\}$ and $B = \{11,12,13\}$.

A number is chosen at random from each set.

What is the probability that the sum of the two numbers selected is greater than or equal to 21?

9 & 12

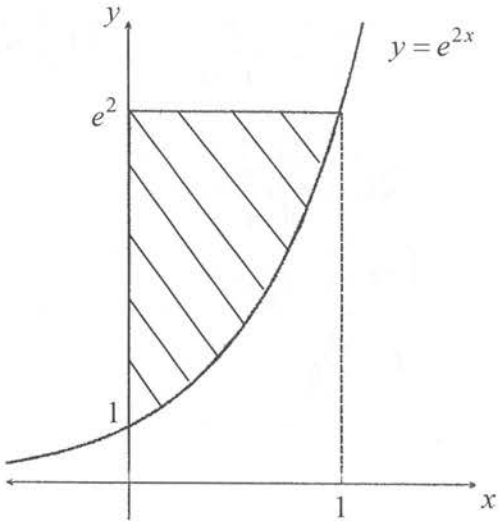
A. $\frac{1}{9}$ $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

B. $\frac{1}{6}$ 9 & 13 +

C. $\frac{2}{9}$ $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

D. $\frac{1}{3}$ $\frac{2}{9}$

4. Which of the following would give the correct value of the shaded area shown in the diagram below?



NOT TO SCALE

$y = e^{2x}$

$\ln y = 2x$

$x = \frac{1}{2}\ln y$

A. $\int_0^1 e^{2x}\,dx$

B. $\int_0^1 \frac{1}{2}\ln x\,dx$

C. $\int_1^{e^2} e^{2y}\,dy$

D. $\int_1^{e^2} \frac{1}{2}\ln y\,dy$

5. What is the domain of the function $f(x) = \ln(3-x) + \sqrt{x-2}$?

$3 - x > 0$ & $x - 2 \geq 0$

$3 > x$ $\quad x \geq 2$

A. $(2,3)$

B. $[2,3)$

C. $(2,3]$

D. $[2,3]$

6. An infinite geometric series has a first term of 12 and a limiting sum of 36.

What is the common ratio?

$S = \frac{a}{1-r} \quad |r| < 1$

$36 = \frac{12}{1-r}$

$1-r = \frac{1}{3}$

$r = \frac{2}{3}$

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

7. What is the correct expression for the indefinite integral $\int \frac{5x}{x^2-3}\,dx$

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$

$f(x) = x^2 - 3$

$f'(x) = 2x$

$\int \frac{5x}{x^2-3}\,dx = \frac{5}{2}\int \frac{2x}{x^2-3}\,dx$

$= \frac{5}{2}\ln|x^2-3| + C$

A. $\frac{2}{5}\ln|x^2-3|+C$

B. $\frac{5}{2}\ln|x^2-3|+C$

C. $2\ln|x^2-3|+C$

D. $\frac{1}{5}\ln|x^2-3|+C$

8. The table below shows the values of the functions $f(x)$ and $g(x)$ for various values of x.

x	1	2	3	4	5
$f(x)$	3	4	5	1	2
$g(x)$	5	3	2	1	4

What is the value of $f(g(4))$?

$g(4) = 1$

A. 1

B. 2

C. 3

D. 4

9. Which transformations listed are required to obtain the graph of $y = x^2 + \frac{1}{2}x - 3$ from the graph of $y = 4x^2 + x$?

A. Horizontal dilation by a factor of 2; vertical translation of 3 units upwards

B. Horizontal dilation by a factor of 2; vertical translation of 3 units downwards

C. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units upwards

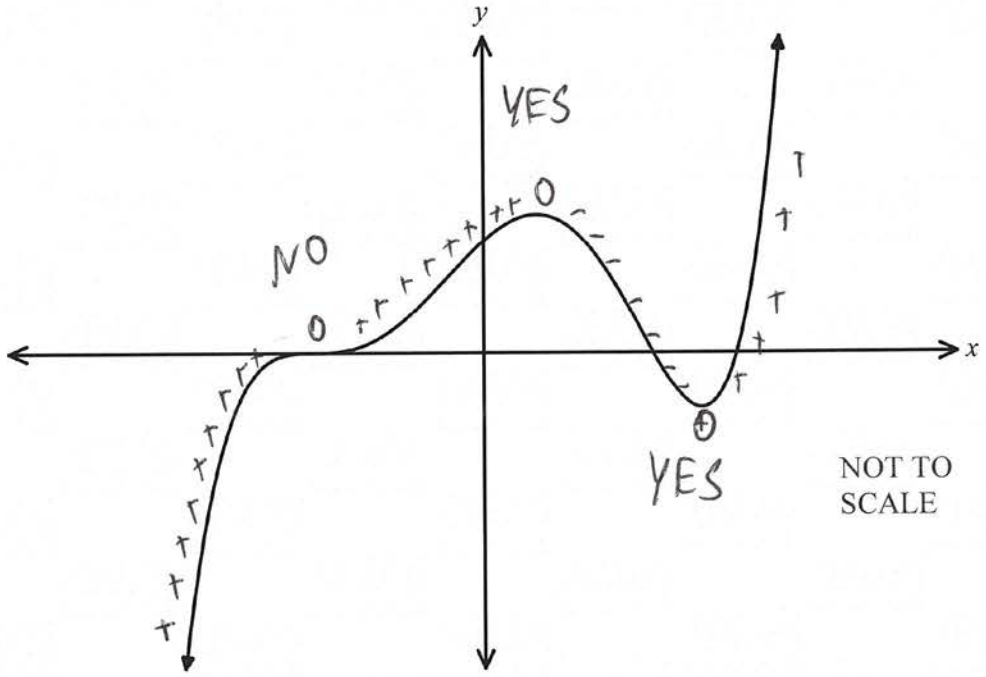
D. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units downwards

$y = 4x^2 + x \xrightarrow{x \to \frac{x}{2}} y = 4\left(\frac{x}{2}\right)^2 + \frac{x}{2} \xrightarrow{y \to y+3} y + 3 = x^2 + \frac{1}{2}x$

$y = x^2 + \frac{1}{2}x - 3$

$x^2 + \frac{1}{2}x$

10. The graph of $y = f'(x)$ is shown below.



How many inflection points does $y = f(x)$ have?

Inflection Point = Place where y'' changes direction.

A. 0

B. 1

C. 2

D. 3

End of Section I

Student Number: | | | | | | | | | |

2023 HSC Course

Assessment Task 4

Mathematics Advanced

Section II
90 marks

Attempt Questions 11-15
Allow about 2 hour and 45 minutes for this section

Answer the questions in the spaces provided.

These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

Question 11 (18 marks) **Marks**

(a) What is the derivative of $\sqrt{3+x^2}$? **2**

$y = (3+x^2)^{1/2}$

$y' = \frac{1}{2}(3+x^2)^{-1/2} \times 2x$ (1) (1)

$= \frac{x}{\sqrt{3+x^2}}$

Question 11 continues on next page

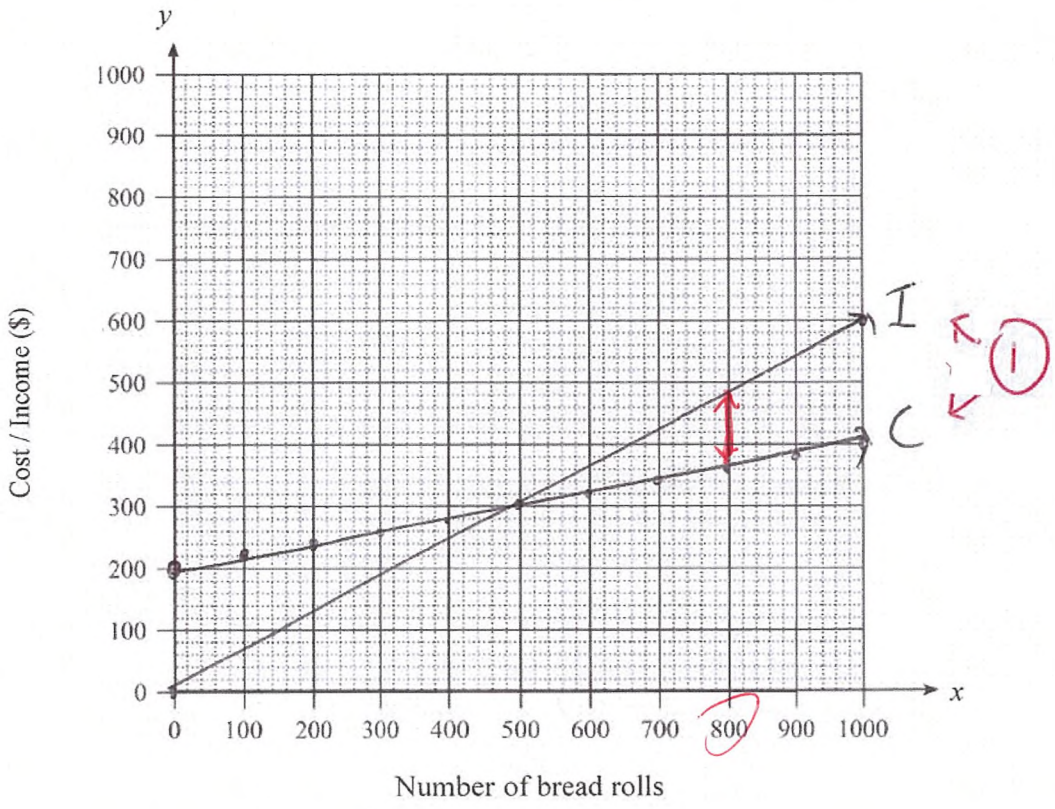
Question 11 (continued)

(b) A bakery shop sells bread rolls for \$0.60 each. The cost, C dollars, of making n bread rolls is given by $C = 200 + 0.2n$.

(i) How much does every 100 bread rolls made add to the cost of production? **1**

$0.2 \times 100 = \$20$ (1)

(ii) On the grid below, draw the graphs of the cost, C, and the income, I. **2**



Hence, find the number of bread rolls that must be sold to break even.

500 (1) f.t.

(iii) How many bread rolls must be sold to make a profit of \$120? **2**

$0.6n - (200 + 0.2n) = 120$ (1)

$0.4n = 320$

$n = 800$ (1)

Question 11 continues on next page

Question 11 (continued)

(c) Let $f(x)=\frac{x}{x^2+1}$.

(i) Find $f'(x)$. 2

$u = x \qquad v = x^2+1$ (1)

$u' = 1 \qquad v' = 2x$

$f'(x) = \frac{(x^2+1)\times 1 - x\times 2x}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2}$ (1)

(ii) Hence, or otherwise, find the equation of the tangent to the graph of $y=f(x)$ at the point where $x=2$, in general form. 3

$f(2) = \frac{2}{2^2+1} = \frac{2}{5} \qquad f'(2) = \frac{1-2^2}{(2^2+1)^2} = \frac{-3}{25}$ (1)

$\left(2, \frac{2}{5}\right)$ (1)

$y - \frac{2}{5} = \frac{-3}{25}(x-2)$

$25y - 10 = -3x + 6$

$3x + 25y - 16 = 0$ (1)

$y = \frac{-3}{25}x + \frac{16}{25}$

Question 11 continues on next page

Question 11 (continued)

(d) The probability distribution of a random variable X is shown below. **1**

x	0	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$2k$	k

What is the value of k?

$k+2k+3k+2k+k=1$

$9k=1$

$k=\frac{1}{9}$ (1)

(e) Given that $\ln a - \ln b = \ln(a-b)$, where $a>b>1$, show that $a=\frac{b^2}{b-1}$. **2**

$\ln\frac{a}{b}=\ln(a-b)$

$\frac{a}{b}=a-b$ (1)

$a=ab-b^2$

$a-ab=-b^2$

$a(1-b)=-b^2$

$a=\frac{-b^2}{1-b}\times\frac{-1}{-1}$ (1)

$=\frac{b^2}{b-1}$

Question 11 continues on next page

Question 11 (continued)

(f) (i) Sketch the graph of $y = 2|x+1|$. **1**

$x = 0$

$y = 2|0+1|$

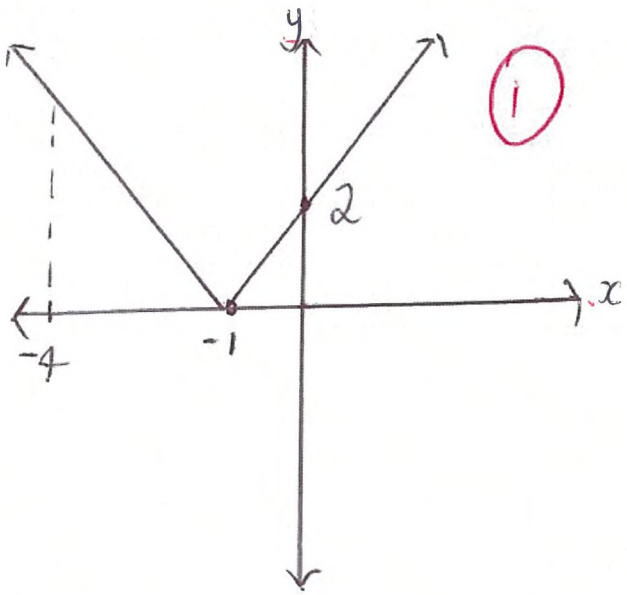
$= 2$

$y = 0$

$0 = 2|x+1|$

$x+1 = 0$

$x = -1$



(1)

(ii) Hence, or otherwise, find $\int_{-4}^{0} 2|x+1|\,dx$. **2**

$x = -4 \quad y = 2|-4+1|$

$= 6$ (1)

Area under curve $= \frac{1}{2} \times 3 \times 6 + \frac{1}{2} \times 1 \times 2$ (1)

$= 10$ units^2

$\therefore \int_{-4}^{0} 2|x+1|\,dx = 10$

End of Question 11

Section II Extra writing space

Student Number: | | | | | | | | | |

Question Number: |

Student Number: | | | | | | | | | |

Question 12 (18 marks)

- For questions in Question 12, your responses should include relevant reasoning and/or calculations.

(a) Let $f'(x) = x^2 + kx$. **3**

The line $y = 2x + 1$ is a tangent to the graph of $y = f(x)$ at the point where $x = -1$.

Find $f(x)$.

As $y = 2x + 1$ is a tangent at $x = -1$

then $f'(-1) = 2$

$(-1)^2 + k \times -1 = 2$

$1 - k = 2$ (1)

$k = -1$

$\int f'(x)\,dx = f(x)$ (1)

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} + C$

OR

Point needed: $x = -1$, $y = 2x - 1 + 1$

$y = -1$

$\therefore f(-1) = -1$ (1)

$f(-1) = -1 = \frac{(-1)^3}{3} - \frac{(-1)^2}{2} + C$

$-1 = -\frac{1}{3} - \frac{1}{2} + C$

$C = -\frac{1}{6}$ (1)

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - \frac{1}{6}$

Question 12 continues on next page

Question 12 (continued)

(b) Helga is training for a marathon. Her training includes a run every Sunday, starting with a run of 3 km on the first Sunday. Each Sunday she increases the length of her run from the previous Sunday by 2 km.

(i) Show that on the 5th Sunday of training, she runs 11 km. **1**

3 (+2) 5 (+2) 7 (+2) 9 (+2) 11 ✓1

$T_n = 3 + (n-1) \times 2$

$T_5 = 11$

(ii) Find a simplified expression for length of her training run on the nth Sunday. **1**

$T_n = 3 + (n-1) \times 2$

$= 2n + 1$ ✓1

(iii) On the nth Sunday, Helga runs 43 km. Find the value of n. **1**

$2n + 1 = 43$

$2n = 42$

$n = 21$ ✓1

(iv) Find the total distance she runs on Sundays, in 15 weeks of training. **2**

$S_{15} = \frac{15}{2}(2 \times 3 + (15-1) \times 2)$ ✓1

$= 255$ ✓1

Question 12 continues on next page

Question 12 (continued)

(c) The frequency distribution table and cumulative frequency distribution table below show the distribution of the heights of trees in a garden. Some of the values are missing.

Height (m)	Frequency		Height less than	Cumulative Frequency
0.1-0.4	2		0.45	2
0.5-0.8	6		0.85	8
0.9-1.2	7		1.25	15
1.3-1.6	11		1.65	26
1.7-2.0	14		2.05	40
2.1-2.4	3		2.45	43

(i) Complete the tables by filling in the missing values. **2**

(1) for 3 correct / for 3 correct that follow from a previous error

(2) for all correct

(ii) What is the modal class? **1**

1.7 - 2.0 (1)

(iii) If a tree is randomly selected, find the probability the height is less than 1.25 m but not less than 0.45 m. **1**

$\frac{15-2}{43} = \frac{13}{43}$ or $\frac{6+7}{43} = \frac{13}{43}$ (1)

Question 12 continues on next page

Question 12 (continued)

(d) Let $f(x) = x^3 - x^2 - x$. Find the stationary points on the graph of $y = f(x)$ and determine their nature. **3**

$f'(x) = 3x^2 - 2x - 1$ (1)

$= (3x+1)(x-1)$

Stationary points when $f'(x) = 0$

$0 = (3x+1)(x-1)$

$x = -\frac{1}{3}$ or $x = 1$

x	-1	$-\frac{1}{3}$	0	1	2
$f'(x)$	4	0	-1	0	7

(1) [or $f''(x)$]

/ — \ — /

MAX MIN

$f(-\frac{1}{3}) = \frac{5}{27}$

$(-\frac{1}{3}, \frac{5}{27})$ MAX T.P.

$f(1) = -1$ (1)

$(1, -1)$ MIN T.P.

Question 12 continues on next page

Question 12 (continued)

(e) Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.55$ and $P(B \mid A) = 0.6$.

(i) Determine whether A and B are independent events. **1**

$P(B|A) \neq P(B)$

$\therefore$ not independent (1)

(ii) Find $P(A \cup B)$. **2**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.55 -$

$P(A \cap B) = P(B|A) \times P(A)$

$= 0.6 \times 0.4$

$= 0.24$ (1)

$P(A \cup B) = 0.4 + 0.55 - 0.24$ (1)

$= 0.71$

End of Question 12

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 13 (18 marks)

- For questions in Question 13, your responses should include relevant reasoning and/or calculations.

(a) Consider the geometric series below. **2**

$$\left(\frac{2}{x+3}\right) + \left(\frac{2}{x+3}\right)^2 + \left(\frac{2}{x+3}\right)^3 + \ldots$$

Show that the series will have a limiting sum when $x = 2$, but not when $x = -2$.

$r = \frac{2}{x+3}$

$x = 2, \; r = \left|\frac{2}{5}\right| < 1$

$\therefore$ limiting sum (1)

$x = -2, \; r = \left|\frac{2}{1}\right| \geq 1$ (1)

$\therefore$ no limiting sum

Question 13 continues on next page

Question 13 (continued)

(b) For what values of x, in the interval $0 \le x \le 2\pi$, does the line $y = -1$ intersect the graph of $y = 2\cos 3x$? **3**

$2\cos 3x = -1$ (1)

$\cos 3x = -\frac{1}{2}$

Neg in Q2, Neg in Q3

$3x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$, (1)

$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3} + 2\pi, \frac{4\pi}{3} + 2\pi, \frac{2\pi}{3} + 4\pi, \frac{4\pi}{3} + 4\pi, \ldots$

$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{20\pi}{3}, \ldots$

$x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \cancel{\frac{20\pi}{9}}$

(1) too big

Question 13 continues on next page

Question 13 (continued)

(c) Consider the function $f(x) = 3x - \sin x$.

(i) Show that $f(x)$ is an odd function. **1**

$f(-x) = 3x - x - \sin(-x)$

$= -3x + \sin x$

$= -(3x - \sin x)$ (1)

$= -f(x)$

(ii) Given that $f(x) \geq 0$ for $0 \leq x \leq \pi$, find the area bounded by the curve $y = f(x)$ and the x-axis, from $x = -\pi$ to $x = \pi$, leaving your answer in exact form. **2**

$A = 2\int_0^{\pi} (3x - \sin x)\, dx$

$= 2\left[\frac{3x^2}{2} + \cos x\right]_0^{\pi}$ (1)

$= 2\left[\left(\frac{3\pi^2}{2} + \cos\pi\right) - (0 + \cos 0)\right]$

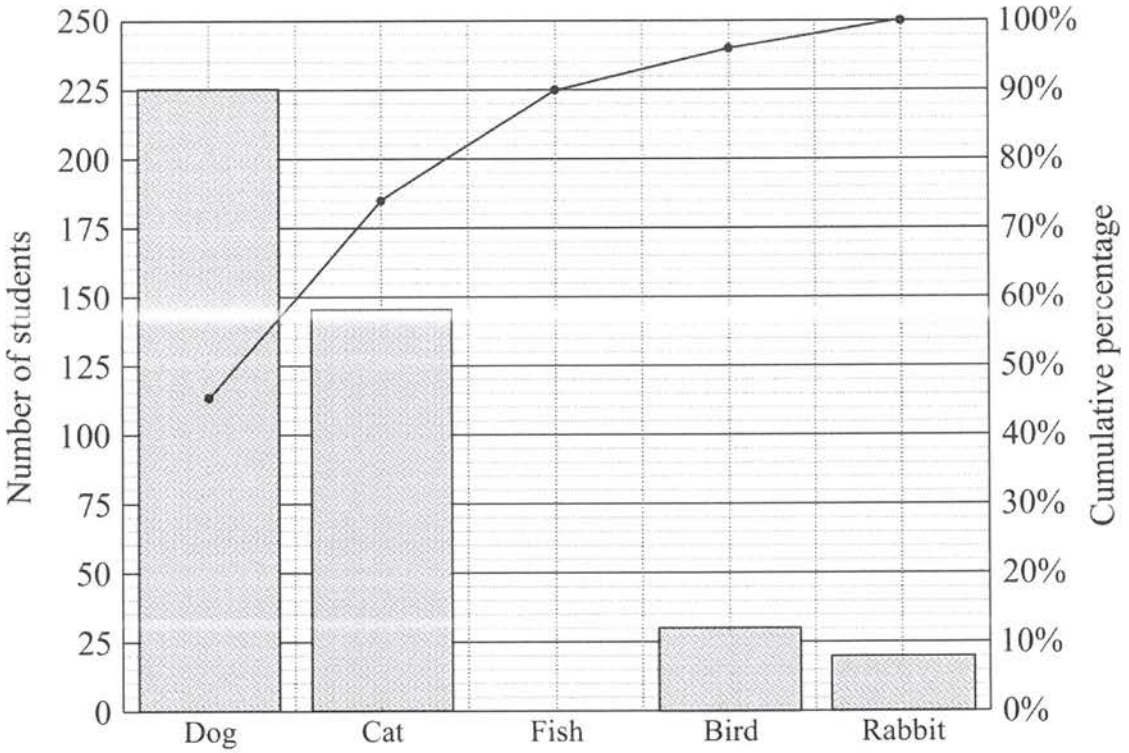
$= 2\left[\frac{3\pi^2}{2} - 2\right]$ (1)

$= 3\pi^2 - 4$ units^2

Question 13 continues on next page

Question 13 (continued)

(d) A group of students was surveyed and the data relating to the types of pets they owned was collected. The Pareto Chart shows the data collected. The column representing the number of students owning a pet fish has been removed.



(i) How many students own a pet dog or cat? **1**

225 + 145 = 370 (1)

(ii) How many students own a pet fish? **2**

74% is 370

1% is 5

100% is 500 (1)

Fish: 90% - 74% = 16% (1)

16% of 500 = 80

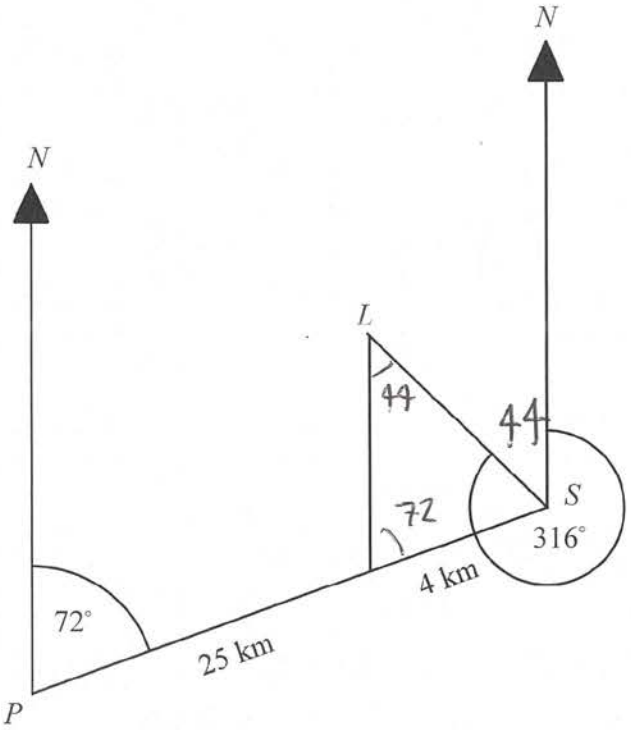
Question 13 continues on next page

Question 13 (continued)

(e) A ship (S) leaves Port (P) travelling on a bearing of 072°.

After travelling 25 kilometres, the ship is due South of a lighthouse (L).

The ship continues on this bearing for a further 4 kilometres, then measures the bearing of the lighthouse to be 316°.



NOT TO SCALE

(i) Show that $\angle PSL = 64°$. **1**

360 - 316 = 44

180 - 72 - 44 = 64

(ii) Calculate the distance LS from the ship to the lighthouse at this time. **2**

Give your answer correct to 1 decimal place.

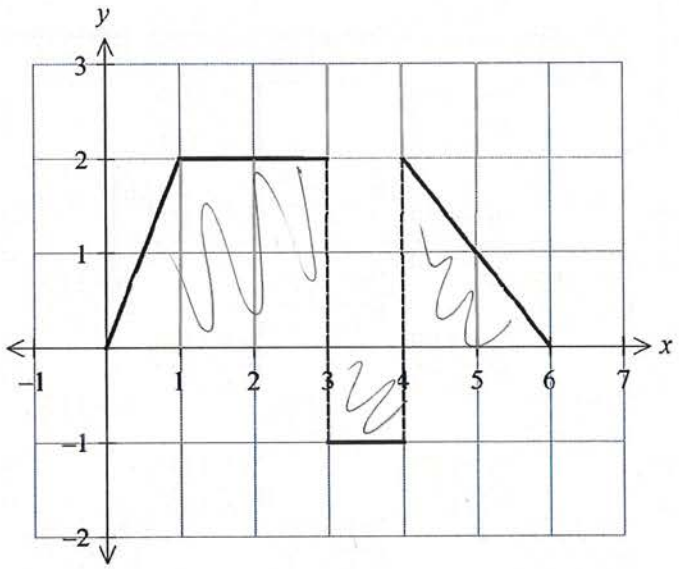
$\frac{LS}{\sin 72} = \frac{4}{\sin 44}$ (1)

$LS = \frac{4 \sin 72}{\sin 44} = 5.5 \text{ km}$ (1)

Question 13 continues on next page

Question 13 (continued)

(f) Consider the function $f(x)$ shown below.



Evaluate the following integrals.

(i) $\int_0^6 f(x)\,dx$ **1**

$\frac{2}{2}(3+2) - 1\times1 + \frac{1}{2}\times2\times2$

$= 6$ (1)

(ii) $\int_0^4 \left[f(x)-2\right]\,dx$ **2**

$\int_0^4 (f(x)-2)\,dx = \int_0^4 f(x)\,dx - \int_0^4 2\,dx$ (1)

$= 4 - [2x]_0^4$

$= 4 - (2\times4 - 0) = -4$ (1)

Question 13f continues on next page

Question 13f (continued)

(iii) $\int_5^6 f'(x)\ dx$ **1**

$\int_5^6 f'(x)dx = [f(x)]_5^6 = f(6) - f(5)$

$= 0 - 1$

$= -1$ (1)

End of Question 13

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 14 (18 marks)

- For questions in Question 14, your responses should include relevant reasoning and/or calculations.

(a) Find $\int \cos x\left(1-\sin^2 x\right)\,dx$. **3**

$\int(\cos x - \cos x \sin^2 x)\,dx$ (1)

$= \int \cos x\,dx - \int \underbrace{\cos x}_{f'(x)}\ \underbrace{\sin^2 x}_{f(x)^n}\,dx$

$= \sin x - \frac{\sin^3 x}{3} + C$

(1) (1)

Question 14 continues on next page

Question 14 (continued)

(b) The sum of the first n terms of an arithmetic series is given by the formula **3**

$S_n = 3n^2 - 17n$, where $n > 0$. Find an expression for the nth term of the series.

$T_n = S_n - S_{n-1}$ (1)

$= 3n^2 - 17n - [3(n-1)^2 - 17(n-1)]$ (1)

$= 3n^2 - 17n - 3[n^2 - 2n + 1] + 17n - 17$

$= 3n^2 - 17n - 3n^2 + 6n - 3 + 17n - 17$

$T_n = 6n - 20$ (1)

Question 14 continues on next page

Question 14 (continued)

(c) The population of parrots, P, is modelled by the function $P = P_0 e^{-kt}$, where t is time in years since May 2004.

In May 2004, there were 2500 parrots and by May 2014 the population had decreased to 1750.

(i) Show that $P_0 = 2500$. **1**

$t=0 \quad P=2500, \quad 2500 = P_0 e^{-k\times 0}$ (1)

$= P_0$

(ii) Find the value of k. (Answer correct to four decimal places). **2**

$t=10 \quad P=1750 \quad = 2500e^{-10k}$ (1)

$\frac{7}{10} = e^{-10k}$

$\ln \frac{7}{10} = -10k$

$k = \frac{\ln 7/10}{-10}$

$= 0.0357$

Question 14c continues on next page

Question 14c (continued)

(iii) If the population continues to decrease in this manner, what will be the expected population in May 2024? Answer to the nearest whole number. **2**

$t = 20$

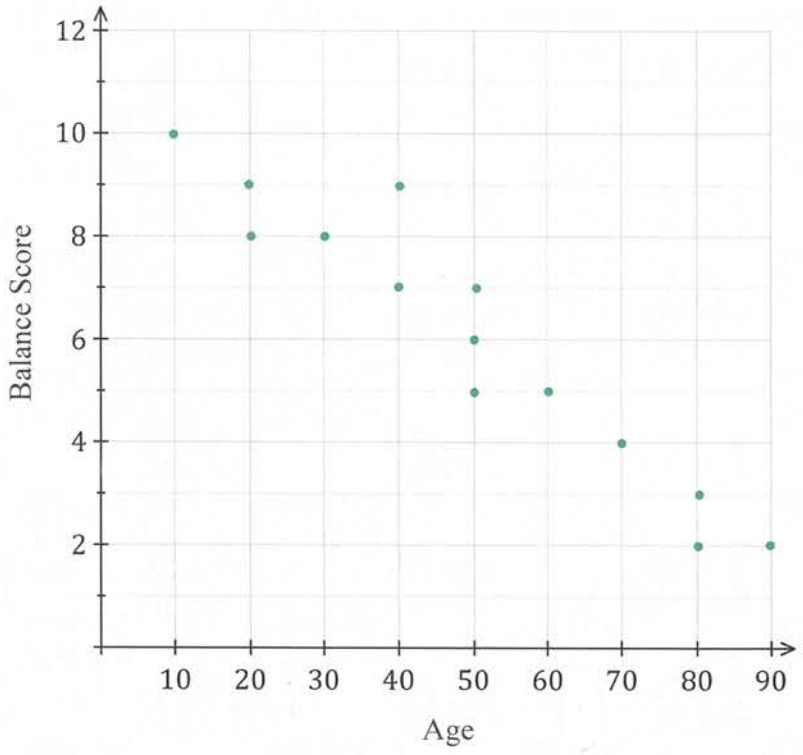
$P = 2500 \times e^{-k \times 20}$ (1)

$= 1225$ parrots (1) [f.t. prev. ans.]

Question 14 continues on next page

Question 14 (continued)

(d) A scatter plot below shows the relationship between Age and Balance Score.



(i) The correlation coefficient is -0.955. Describe the association between Age and Balance Score with reference to the correlation. **1**

Strong negative correlation (1)

(ii) The least squares regression line for this data is $y = 11.1249 - 0.1025x$. **1**

Using this regression line, predict the Balance Score of a 65 year old.

$y = 11.1249 - 0.1025 \times 65$

$= 4.4624$ (1)

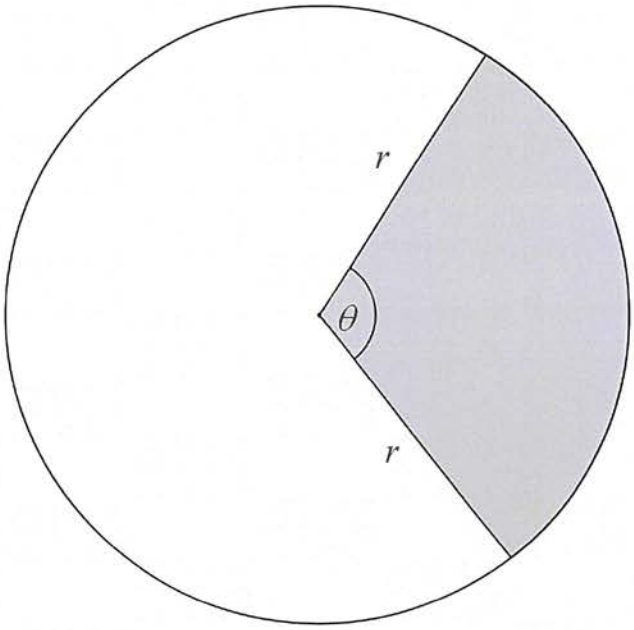
(iii) Comment on whether your answer in part (ii) is reliable. **1**

Yes as strong correlation & interpolation. (1)

Question 14 continues on next page

Question 14 (continued)

(e) The diagram below shows a shaded sector in a circle with radius r and centre O. The sector subtends an angle of θ at the centre of the circle. The area of the sector is 49 cm^2. **4**



NOT TO SCALE

Show that $\theta = \frac{98}{r^2}$ and hence show that the minimum perimeter of the sector will occur when the arc length of the sector is equal to the diameter of the circle.

$49 = \frac{1}{2} r^2 \theta$ (1)

$98 = r^2 \theta$

$\theta = \frac{98}{r^2}$

$P = 2r + r\theta$

$= 2r + r \times \frac{98}{r^2}$

$P = 2r + 98r^{-1}$ (1)

$\frac{dP}{dr} = 2 - 1 \times 98 \times r^{-2}$

$= 2 - \frac{98}{r^2}$

Question 14e continues on next page

Question 14e (continued)

Turning point when $\frac{dP}{dr} = 0$

$0 = 2 - \frac{98}{r^2}$

$0 = 2r^2 - 98$

$r^2 = 49$

$r = \pm 7$

$= 7$ as $r > 0$ (1)

$\frac{d^2P}{dr^2} = -2 \times -1 \times 98 \times r^{-3}$

$\frac{d^2P}{dr^2} = 196r^{-3}$

when $r = 7$

$\frac{d^2P}{dr^2} = 0.57... > 0$

$\therefore r = 7$ is when P is MIN

Arc length $= 7 \times \frac{98}{7^2} = 14 = 2 \times 7$ (1)

$\therefore$ diameter = arc length, for MIN perimeter.

End of Question 14

Section II Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Question 15 (18 marks)

- For questions in Question 15, your responses should include relevant reasoning and/or calculations.

(a) As a particle moves, its velocity, in metres per second, is described by the equation

$$v(t) = -2t^2 + 2t + 4$$

where $t \geq 0$ is the time in seconds. The particle is initially 4 metres right of the origin.

(i) Find the time taken for the particle to reach its maximum velocity. **2**

Concave down parabola so max occurs at turning point. (1)

$v' = -4t + 2$

$0 = -4t + 2$

$t = \frac{1}{2}$ (1)

(ii) Find the position of the particle when it first comes to rest. **3**

$v(t) = 0 = -2t^2 + 2t + 4$

$= -2(t^2 - t - 2)$

$0 = -2(t-2)(t+1)$ (1)

$t = 2$ or $t = -1$

$t = 2$ as $t \geq 0$

$x(t) = \int v\,dt = \frac{-2t^3}{3} + t^2 + 4t + C$ (1)

$t = 0, x = 4 \therefore C = 4$

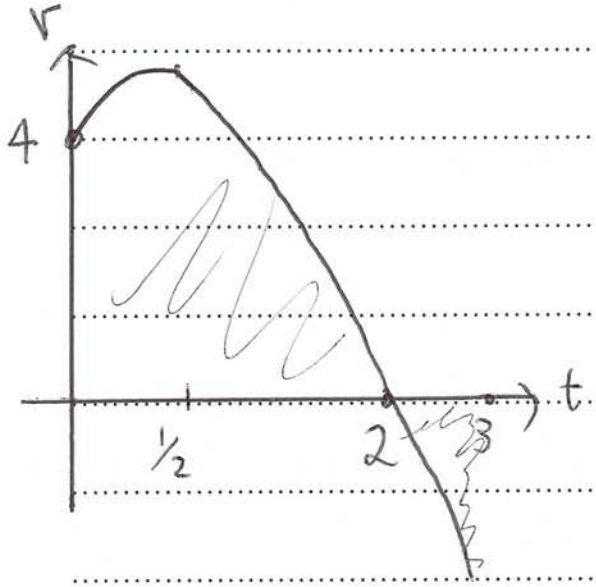
$x(2) = \frac{-2 \times 2^3}{3} + 2^2 + 4 \times 2 + 4 = \frac{32}{3}$ m (1)

Question 15a continues on next page

Question 15a (continued)

(iii) Find the distance travelled by the particle in the first 3 seconds. **2**

Distance travelled = total area under the curve.



$$\int_0^2 v\,dt + \left| \int_2^3 v\,dt \right| \quad (1)$$

$$\left[-\frac{2}{3}t^3 + t^2 + 4t\right]_0^2 + \left| \left[-\frac{2}{3}t^3 + t^2 + 4t\right]_2^3 \right|$$

$$= \frac{20}{3} - 0 + \left| 3 - \frac{20}{3} \right| \quad (1)$$

$$= \frac{31}{3} \text{ m.}$$

Question 15 continues on next page

Question 15 (continued)

(b) The point $A(4,21)$ lies on the graph of $y=g(x)$. **3**

It is known that the graph of $y=g(x)$ is obtained from transforming the graph of $y=f(x)$ such that $g(x)=-2f(4x+4)+1$.

Find the coordinates of the point on the graph of $y=f(x)$ which is mapped to the point A.

To get from $f(x) \rightarrow g(x)$

H.D. $\frac{1}{4}$

H.T. Left 1 (1)

V.D. -2

V.T. up 1

Reverse for $(4,21)$

Down 1 $\downarrow$

$(4,20)$

V.D. $-\frac{1}{2}$ $\downarrow$ (1)

$(4,-10)$

Right 1 $\downarrow$

$(5,-10)$

H.D. 4 $\downarrow$

$(20,-10)$ (1)

Alternative.

$g(4) = 21 = -2 \times f(4\times4+4)+1$ (1)

$21 = -2f(20)+1$

$20 = -2f(20)$ (1)

$-10 = f(20)$

$(20,-10)$ (1)

Question 15 continues on next page

Question 15 (continued)

(c) The function $y = f(x)$ is defined as 2

$$f(x) = \begin{cases} 1 & \text{for} \quad -3 \le x \le 0 \\ -x^2 + 1 & \text{for} \quad x > 0 \end{cases}$$

Determine whether $y = f(x)$ is both continuous **and** differentiable at $x = 0$.
Justify your answer with appropriate calculations.

$y=1$ & $y=-x^2+1$ are both continuous and differentiable for all real x.

$x=0$, $f(0)=1$ and $-x^2+1 = -0^2+1 = 1$ (1)

$\therefore$ continuous

$x=0$

$f'(0)=0$ $\quad y=-x^2+1$

$y'=-2x$

$x=0$

$y'=0=f'(x)$

$\therefore$ differentiable. (1)

Question 15 continues on next page

Question 15 (continued)

(d) On a given day, the height of the water in a river is modelled by the function

$$h(t) = 5 + 2\sin\left(\frac{\pi t}{4}\right),$$

where h is the height of the water, in metres, and t is the time, in hours, after 12 am.

(i) What is the height of the water at 12 am? **1**

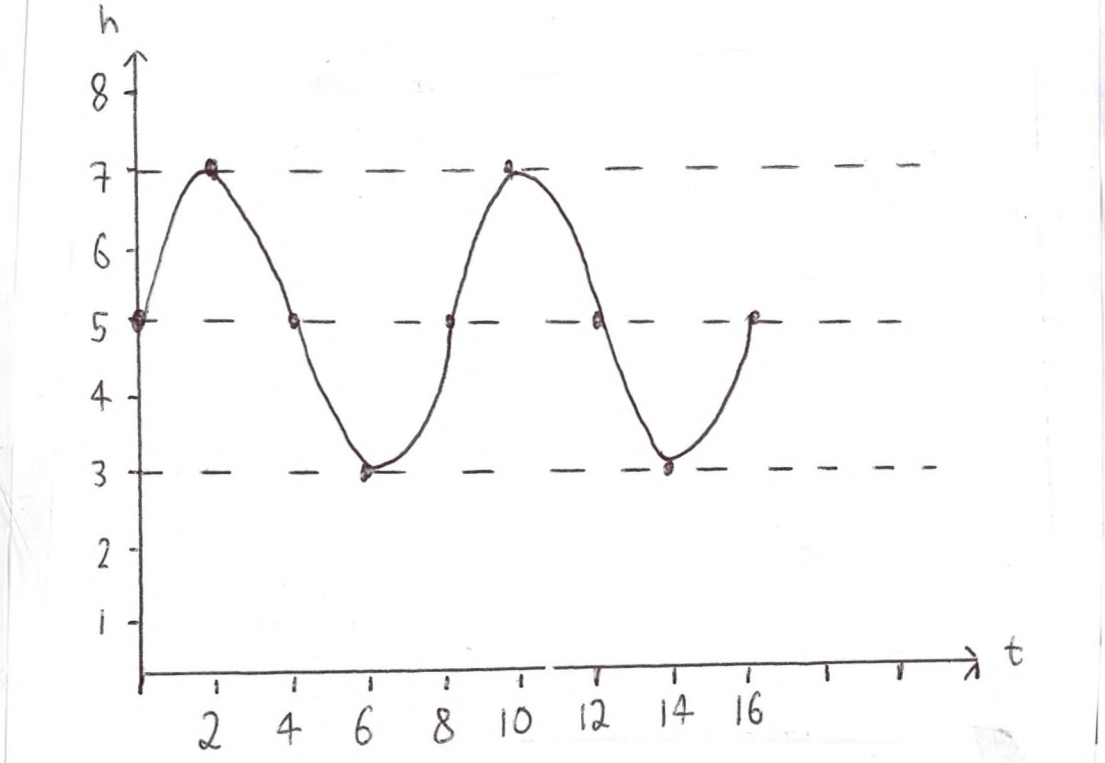
$h(0) = 5 + 2\sin(0)$

$= 5\text{m}$ (1)

(ii) Sketch the graph of $h(t) = 5 + 2\sin\left(\frac{\pi t}{4}\right)$, in the domain $[0,16]$. **2**

Repeats when $\frac{\pi t}{4} = 2\pi$

$t = 8$ = Period



(1) Vertical transformation

(1) Horizontal transformation

Question 15d continues on next page

Question 15d (continued)

(iii) A family decides to go on a picnic by the river from 12 pm to 2 pm. **3**
It is only safe to swim in the river if the height of the water is less than 4 metres.

When is the earliest time the family can swim in the river after 12 pm?
Give your answer correct to the nearest minute.

$5 + 2\sin\left(\frac{\pi t}{4}\right) = 4$

$\sin\left(\frac{\pi t}{4}\right) = -\frac{1}{2}$ (1)

S	A
T	C

$\frac{\pi t}{4} = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6} \ldots$ (1)

$t = \frac{28}{6}, \frac{44}{6}, \frac{76}{6}, \frac{92}{6}$

as $12 \leq t \leq 14$

$t = \frac{76}{6} = 12:40$ pm (1)

End of Paper